Homework 8

Using the assigned reading listed on the course page, answer the questions below with a short response. Note that we are looking for concise statements that show understanding, not quantity. The total discussion should roughly be a page.

Graphs and geometry

1. A graph neural network G_θ operates on the vertices V and edges E by propagating information across the neighboring nodes.

   (a) What invariance assumptions does a graph neural network make?

   (b) Explain the relationship between graph neural networks and convolution operators? In what sense are they similar and dissimilar?

   (c) Explain how one can formulate an image classification problem as a graph neural network? What are computational and memory issues associated with this formulation? Provide some ideas to alleviate these issues?

2. Graph neural networks often operate on the Graph Laplacian L instead of the adjacency matrix. The Graph Laplacian is defined as $L = D - A$, where D is the degree matrix that tells us how many edges are connected to the nodes, and A is the adjacency matrix that tells us how the nodes connected. (For background on Graph Laplacians see Section IV of the assigned Bronstein et al. reading and https://en.wikipedia.org/wiki/Laplacian_matrix.)

   (a) Graph Laplacians have some interesting properties that make it a good representation of the graph to train neural networks on top of. What does taking n-th power of L do?
   (**Hint**: construct a small graph. Compute L and multiply it by some one-hot x and see what information Lx contains. What about L^2x?)

   (b) For a set of nodes x and the updated output nodes y and learnable parameters w_i, we define a polynomial graph layer as:

   $$y = \sum_{i=0}^{d} w_i L^i x \tag{1}$$

   What is the intuition of this polynomial layer, and how is it related to convolutions? You can use part (a) to explain what this layer is doing.
   (**Hint**: first set everything to zero except $w_0 = 1$, now what about $w_1 = 1$?)

Submission: Upload a PDF of your response through Canvas by **11/9 at 1pm**.